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# Hawking Temperature Modification and the Physical Dynamics of Black Holes: A Study of the Influence of Internal and Cosmological Variables

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***ABSTRACT***
*The main objective of the study is to develop a model that modifies the traditional Hawking temperature by considering the influence of internal variables such as radius, mass, electric charge, angular momentum, and cosmological constant. The research method involves mathematical analysis and computational modeling based on the modified Hawking temperature equation. The results show that the modified Hawking temperature produces non-linear corrections that show the interaction between black holes and the quantum structure of spacetime. graphical representations visualize the variation of Hawking temperature with changes in area, electric charge, angular momentum, and cosmological parameters. The implications of the research extend to the understanding of the thermal properties of black holes in the context of gravitational and quantum theories. The research identifies gaps in the knowledge of the effects of cosmological parameters on black hole thermodynamics and introduces Hawking temperature modifications that have not been mapped in detail before. The study concludes that the Hawking temperature modification provides a strong foundation for further research in black hole physics, particularly in the effect of physical and cosmological parameters on the thermal properties of black holes.*

## INTRODUCTION

Black holes are among the most mysterious objects in astrophysics that have challenged our understanding of the universe [1] [2]. Black Holes are formed from the collapse of supermassive stars that produce gravitational fields so strong that no matter or even light can escape their gravity after crossing a boundary called the "event horizon" [3] [4]. The consequence of the existence of an event horizon is that the black hole becomes an object that appears to be 'black' in the sense that it does not emit any light [5] [6].

The history of understanding black holes has evolved rapidly along with advances in Albert Einstein's general theory of relativity in the early 20th century [7]. The concept of black holes as solutions to Einstein's field equations was first coined in 1916, but the initial understanding of them was still in relation to the relatively simple theory of classical gravity [8]. A significant turning point in the study of black holes came in 1974 when Stephen Hawking introduced the concept of Hawking radiation [9]. Stephen Hawking points out that black holes should not actually be 'black' as they can undergo a process of emitting tiny sub-atomic particles called 'Hawking radiation' [10]. This process is triggered by quantum effects near the event horizon of a black hole, where a pair of particle-antiparticles emerging from the vacuum can separate before one falls into the black hole, while the other detaches and leaves the black hole [11]. Hawking radiation plays an important role in modifying our understanding of the thermodynamic properties of black holes [12]. Black holes are actually impossible to reach absolute zero temperature, as described by the second law of thermodynamics, but they can have measurable temperatures related to the particle emission process [13].

Recent understanding of black holes has expanded the concept by considering quantum effects and their relationship with more fundamental theories of physics, such as quantum field theory and quantum theory of gravity [14] [15]. This makes black holes not only fascinating astrophysical objects, but also testing grounds for sophisticated physical theories. The development of black hole theory, including the understanding of Hawking radiation, has become one of the most exciting and challenging areas of research in modern astrophysics, with the potential to change our paradigm of the universe.

The research aims to deepen the understanding of black holes through the modification of the Hawking temperature equation. Research aims to improve the accuracy and complexity of understanding the thermal dynamics of black holes. Research aims to explain how these physical parameters interact with the cosmological environment and the surrounding quantum field. It is hoped to clearly understand the complex relationship between the thermal properties of black holes and their physical and environmental characteristics. The benefits of the research are vast, including a better understanding of the properties of black holes in their thermal and dynamical context. Potential implications of the research results include significant contributions to the study of cosmology and quantum physics.

The research limitations focus on the development and theoretical analysis using the modified Hawking temperature equation. The research aims to understand how various physical parameters such as black hole mass, radius, electric charge, angular momentum, and cosmological constant affect the thermal properties of black holes. The modified Hawking temperature equation (Equation (29)) becomes the main theoretical basis used to explain the complex interactions. The research is further complemented by mathematical analysis using equations (15) and (28) to produce the modified equation (29), and explain the non-linear corrections to the Hawking temperature. However, the research did not involve direct validation using observational or experimental data. While it is important to explain theoretical phenomena, validation with empirical data is a major shortcoming in the study. The limitation of the study lies in its limited focus on theoretical and mathematical analysis. While it can provide an in-depth understanding of the properties of black holes from a theoretical point of view, the research cannot capture the complexities and variations that may exist in direct observations in the field or experiments. Therefore, to confirm the accuracy and relevance of its theoretical results in the real world, additional steps are needed to collect and analyze observational data.

The implications of the research have the potential to change the way we view black holes and existing theories of gravity. Introducing non-linear corrections in the Hawking temperature equation, the research not only tests the limits of conventional gravity theories, but also shows that quantum field effects around the event horizon of a black hole can significantly affect its thermal properties.

The findings have great potential to lead to the revision or development of a more comprehensive theory of gravity, paving the way for new explorations in fundamental physics. The research has a direct impact on future astronomical observations. Considering the new effects revealed by the research, astronomers can design new observations and experiments to test and validate the predictions of newly developed theoretical models. The new theoretical framework emerging from the research has implications in cosmology. Incorporating parameters such as electric charge, angular momentum, and the cosmological constant in the modified Hawking temperature model, research develops an understanding of cosmological evolution. The research not only provides a deeper understanding of the thermal properties of black holes, but illustrates the complexities involved in the interaction of black holes with their cosmic environment. The implications encourage further research in both theoretical and observational physics. Researchers can develop more sophisticated and accurate models to predict black hole behavior in more complex scenarios, expanding the boundaries of our knowledge of the universe.

A research gap is the lack of in-depth studies on the nuanced effects of parameters such as electric charge, angular momentum, and cosmological constant on black hole thermodynamics. Most studies tend to focus on simpler models or do not consider in detail the complex interactions between these parameters and the thermal properties of black holes. The novelty of the research lies in the proposition of a modified Hawking temperature equation that incorporates these variables. This results in a deeper theoretical understanding of black hole physics beyond the existing classical predictions. Incorporating electric charge, angular momentum, and cosmological constants into the model, the research broadens the scope of our knowledge of how black holes interact with their environment, both from a quantum mechanical and cosmological perspective [16]. The research not only closes gaps in current knowledge but also offers new insights that could influence the direction of development of new theories in black hole physics and cosmology.

## METHOD

In black hole thermodynamics, there is an equation that describes the change in black hole mass as a result of changes in horizon area, electric charge, angular momentum, and vacuum energy [17] [18]. The equation is:

$$dM = \frac{\kappa}{8\pi} dA - \Phi dQ - \omega dJ + V d\rho\Lambda \quad (1)$$

In equation (1), (dM) represents the change in mass of the black hole, (dA) is the change in horizon area, (dQ) is the change in electric charge, (dJ) is the change in angular momentum, and (dJ) is the change in angular momentum. $d\rho\Lambda$ is the change in vacuum energy. Each parameter in the equation has a specific definition. Parameter ($\kappa$) is known as surface gravity and is defined by [19]:

$$\kappa = \frac{r-M}{R^2} - \frac{2\Lambda r}{3} \quad (2)$$

Where (r) is the horizon radius, (M) is the mass of the black hole, (R) is the radial distance, and ($\Lambda$) is the cosmological constant. Parameters ($\Phi$) represents the electric potential and is defined by [20]:

$$\Phi = \frac{Qr}{R^2} \quad (3)$$

Where (Q) is the electric charge of the black hole. ($\omega$) represents angular velocity and is defined by [21]:

$$\omega = \frac{a}{R^2} \quad (4)$$

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Where $(a)$ is the angular momentum per unit mass. The parameter (V) represents the thermodynamic volume and is defined by the formula [22]:

$$V = \frac{4}{3}\pi r R^2 \tag{5}$$

Some of the specific parameters above are substituted into the equation dM:

$$dM = \left\{\frac{24\pi(r-m)-16\Lambda r R^2\pi}{R^2 192\pi^2}\right\} dA - \left\{\frac{Qr}{R^2}\right\} dQ + \left\{\frac{a}{R^2}\right\} dJ + \left\{\frac{4}{3}\pi r R^2 \Lambda\right\} d\rho \tag{6}$$

The equations show that the mass of the black hole changes along with changes in other thermodynamic parameters [23]. These parameters affect the mass of the black hole through their individual contributions.

The change in horizon area (dA) is related to the change in mass (dM) [24]. Surface gravity component ($\kappa$ ) involved in the change of horizon area is expressed as $\{24\pi(r-m)/R^2 192\pi^2\}$. There is a component involving the cosmological constant (Λ) which affects the change in mass through the volume generated by the change in horizon area, namely $(16\Lambda r R^2\pi/R^2 192\pi^2)$ [19]. The change in electric charge (dQ) affects the mass of the black hole [25]. Electric potential (Φ) expressed as $(Qr/R^2)$, which reduces the mass of the black hole when there is a change in the electric charge [26]. The change in angular momentum (dJ) is related to the change in mass (dM) [27]. Angular velocity $(\omega)$ expressed as $(a/R^2)$, which increases the mass of the black hole as the angular momentum increases [28]. Change in vacuum energy $(d\rho)$ affects the mass of the black hole [29]. The thermodynamic volume (V) is expressed as $\left({}^4\!/_3\, \pi r R^2 \Lambda\right)$, which increases the mass of the black hole when the vacuum energy increases [30]. The first law of thermodynamics generally states the change in energy in a system (dE) is the result of heat energy added to the system $(\delta Q)$minus the work done by the system $(\delta W)$[31] [32] [33]:

$$dE = \delta Q - \delta W \tag{7}$$

In the general theory of relativity, mass and energy are equivalent according to Einstein's famous equation [34]:

$$E = mc^2 \tag{8}$$

For black holes, it can be stated that the change in mass (dM) is directly related to the change in energy (dE) of the black hole system [35] [36]:

$$dE = d(Mc^2) \tag{9}$$

$$dE = c^2 dM \tag{10}$$

$$c^2 dM = \delta Q - \delta W \tag{11}$$

$$dM = \frac{\delta Q - \delta W}{c^2} \tag{12}$$

In black hole thermodynamics, it is generally considered that (c) as a unit *(by normalizing natural units),* so the equation becomes simpler [37]:

$$dM = \delta Q - \delta W \tag{13}$$

So from the above equation, it can be explained that the change in black hole mass (dM) is the result of changes in heat energy ($\delta Q$) minus the change in the work done by the black hole ($\delta W$), in accordance with the first law of thermodynamics for black holes [38]. ($\delta Q$) from the equation ($dM = \delta Q - \delta W$), one can add the change in effort to the change in mass (dM) in the equation:

$$\delta Q = dM + \delta W \tag{14}$$

To get the concrete equation, substitute the dM equation, so the equation is as follows:

$$\delta Q = \left[\left\{\frac{24\pi(r-m)-16\Lambda r R^2\pi}{192R^2\pi^2}\right\} dA - \left\{\frac{Qr}{R^2}\right\} dQ + \left\{\frac{a}{R^2}\right\} dJ + \left\{\frac{4}{3}\pi r R^2 \Lambda\right\} d\rho\right] + \delta W \tag{15}$$

The above equation states that the infinitesimal change in heat ($\delta Q$) can be expressed as the sum of contributions from various physical parameters and work changes ($\delta W$). Each component in parentheses represents contributions from horizon area (dA), electric charge (dQ), angular momentum (dJ), and vacuum energy (dρ) [39].

The first component relates to the change in horizon area (dA), represented by the coefficient in parentheses $\{24\pi(r-m) - 16\Lambda r R^2\pi\}/192R^2\pi^2$. The coefficient depends on the horizon radius (r), the mass of the black hole (m), and the cosmological constant (Λ), represents how changes in horizon area affect heat [40]. The second component relates to the change in electric charge (dQ), represented by the coefficient of $(Qr/R^2)$. The coefficient is an electric potential that shows the relationship between the change in electric charge (dQ) and the change in heat ($\delta Q$) [41]. The third component relates to the change in angular momentum (dJ), represented by the coefficient of $(a/R^2)$. The coefficient is angular velocity which shows how the change in angular momentum (dJ) affects heat (dQ) [42]. The fourth component relates to the change in vacuum energy (dρ), represented by the coefficient $\left({}^4\!/_3\,\pi r R^2 \Lambda\right)$. The coefficient shows the relationship between the change in vacuum energy (dρ) and the change in heat ($\delta Q$) [43]. ($\delta W$) represents the change in external work that may be done on or by the system [44]. This suggests that in addition to changes in internal parameters such as horizon area, charge, angular momentum, and vacuum energy, changes in heat ($\delta Q$) can also be affected by external work changes ($\delta W$) [45]. The area of the black hole horizon, (A), is given by the formula [46]:

$$A = 4\pi {r_s}^2 \tag{16}$$

Where ($r_s$) is the Schwarzschild radius of the black hole, which is related to the mass of the black hole through the formula [47]:

$$r_s = \frac{2GM}{c^2} \tag{17}$$

where (G) is Newton's gravitational constant. For infinitesimal changes in horizon area (dA), The horizon area differential formula can be used:

$$dA = \frac{d(4\pi {r_s}^2)}{dM} \tag{18}$$

Because ($r_s$) depends on (M), then it can use the chain rule to calculate its differential:

$$dA = 8\pi r_s \frac{dr_s}{dM} dM \tag{19}$$

As for $r_s = 2GM/c^2$, will be obtained:

$$\frac{dr_s}{dM} = \frac{2G}{c^2}$$
$$dA = 8\pi\left(\frac{2GM}{c^2}\right)\left(\frac{2G}{c^2}\right)dM$$
$$dA = \frac{8\pi G^2}{c^4}MdM \tag{20}$$

From the above equation, substitute the dM equation, so that it will be obtained by connecting the Black Hole thermodynamics:

$$dA = \left\{\frac{GM(r-M)}{c^4R^2} - \frac{0.6666.GMr\Lambda}{c^4}\right\}dA - \left\{\frac{Qr}{R^2}\right\}dQ + \left\{\frac{a}{R^2}\right\}dJ + \left\{\frac{4}{3}\pi rR^2\Lambda\right\}d\rho \tag{21}$$

This shows that changes in the mass of the black hole affect not only the mass itself but also the geometric characteristics of the horizon area [5] [48]. The change in electric charge (dQ) affects the horizon area (dA) in the opposite direction, where the change in electric charge (dQ) can decrease the horizon area depending on the charge size (Q), horizon radius (r), and radial distance (R) [25] [49]. The change in angular momentum (dJ) contributes to the change in horizon area (dA), which depends on the angular momentum per unit mass (a) and radial distance (R) [50]. This shows that changes in the black hole's angular momentum affect the geometric size of the black hole [51]. Change in vacuum energy ($d\rho\Lambda$), which is affected by the cosmological constant ($\Lambda$), contributes to the change in horizon area (dA). This indicates that the vacuum energy conditions around the black hole can affect the geometric characteristics of the black hole [19].

Black hole entropy is thought to be related to the microstatic number *(number of possible ways)* of quantum configurations associated with the black hole [52]. One mathematical formulation is that it is proportional to the size of the event horizon [53]. According to the laws of black hole thermodynamics [54]:

$$dE = TdS \tag{22}$$

In classical black hole mechanics, the event horizon area (A). Where ($r_s$) is the Schwarzschild radius of the black hole, which is related to the mass (M) [55] [56] [57]. To derive the relationship between entropy (S) and event horizon area (A), Considering that entropy is defined by [58]:

$$S = \frac{k_B A}{4{l_p}^2} \tag{23}$$

Where $l_p = \sqrt{\hbar G/c^3}$ is the Planck length, and ($\hbar$) is Planck's constant. Hawking temperature ($T_H$) of a black hole is given by the formula:

$$T_H = \frac{\kappa}{2\pi} \tag{24}$$

Entropy ($\delta S$) of the black hole can be related to the change of the black hole event horizon area ($\delta A$) through the laws of black hole thermodynamics [59]:

$$\delta S = \frac{1}{4}\frac{\delta A}{{l_p}^2} \tag{25}$$

Substitution ($T_H$) and ($\delta S$) into the heat energy change equation ($\delta Q$):

$$\delta Q = T_H \delta S \tag{26}$$
$$\delta Q = \left(\frac{\kappa}{2\pi}\right)\delta S$$

Heat energy changes ($\delta Q$) of a black hole expressed as a product of the Hawking temperature ($T_H$) and black hole entropy change ($\delta S$) [12]. Equation obtained equation ($\delta S$) is by dividing the change in heat energy by Hawking Temperature, thus obtaining the equation:

$$\delta S = \frac{\delta Q}{T_H} \tag{27}$$

From the above equation can be done substitution ($\delta Q$) and ($T_H$), so as to get the equation:

$$\delta S = \left[\left\{\frac{3(r-m)}{4} - \frac{\Lambda r R^2}{2}\right\} dA - \{6\pi Q r\} dQ + \{a 6\pi\} dJ + \left\{\frac{32\pi^2 R^4 r \Lambda}{3}\right\} d\rho\right] + \delta W \tag{28}$$

It can be seen how entropy varies ($\delta S$) affected by changes in black hole surface area ($dA$). Term $\{3(r-m)/4 - \Lambda r R^2/2\}dA$ shows that the difference between the black hole's radius (r) and its mass (m) affects the change in surface area. The larger this difference, the greater the contribution to the change in entropy. Cosmological constant ($\Lambda$) plays an important role. The cosmological constant relates to the dark energy that fills the universe, and its effect on the surface area of a black hole is shown by ($\Lambda r R^2/2$)[60]. Term shows how changes in spatial structure and cosmic energy affect black hole entropy [61]. Term $\{(6\pi Q r)dQ\}$ describes how a black hole's electric charge (Q) interacts with its radius (r) to affect its entropy [49]. The change in charge (dQ) directly contributes to the variation in entropy. This is important because the electric charge can affect the electromagnetic field around the black hole, which in turn affects its thermodynamic properties [62]. Term $\{(6\pi a)dJ\}$ shows the contribution from angular momentum (J). Angular momentum is related to the rotation of the black hole, which produces additional gravitational and inertial effects [63]. The parameter (a) associated with angular momentum shows how rotation affects entropy, where the faster the black hole rotates, the greater its contribution to the entropy variation. Cosmological constant ($\Lambda$) reappears in the term $\{(32\pi^2 R^4 r\Lambda)/3\}$. Term shows how the dark energy associated with ($\Lambda$) interacts with the black hole radius and scale parameter (R). This indicates that changes in the additional parameters ($\rho$) which may be related to other cosmological properties, such as energy density, also affect black hole entropy [64] [65]. ($\delta W$) includes contributions from variations in external work or other energy not covered by the previous terms.

## RESULTS

*Hawking Temperature Modification*

In the method section, as seen in Equation (26), substitutions can be made using Equations (15) and (28). The result of the substitution is the modified Hawking temperature equation, which is as follows:

$$T_{BH} = \left[\left\{\frac{3(r^2+M)}{4\pi R^2}\right\} - \left\{\frac{1}{2\pi}\left(\frac{3rM}{R^2} + r\Lambda(r-M) + \pi r(r-m)\right)\right\} + 0.333 r^2 R^2\right] dA - \left[\frac{6\pi Q^2 r^2}{R^2}\right] dQ + \left[\frac{6a^2\pi}{R^2}\right] dJ + [14.2\pi^2 r^2 R^6 \Lambda^2] d\rho + 1 \tag{29}$$

Analysis of the equations reveals various important aspects in understanding the nature of black holes and the associated space-time dynamics. First, $3(r^2+M)/4\pi R^2$ shows the contribution of radius r and mass M to the thermal entropy of the system. Thermal entropy is related to the complexity of the black hole system and shows how the size and mass of the black hole affect its thermal properties [66]. The quantity R refers to a scale or geometry constant that affects the distribution of entropy in space-time. Second, $\{1/2\,\pi(3rM/R^2 + r\Lambda(r-M) + \pi r(r-M))\}$ provides a more complex correction to the Hawking temperature. The correction involves the influence of mass M and cosmological parameters $\Lambda$. The deviation comes from quantum field effects around the black hole event horizon, suggesting a deeper interaction between the black hole and the quantum fabric of space-time [67]. Third, $0.333r^2R^2$ This suggests the presence of significant non-linear effects in the black hole space-time geometry, which cannot be explained by classical gravitational theory. The corrections are indicative of the dynamical properties and structural complexity of black holes. Fourth, $[6\pi Q^2 r^2/R^2]dQ$ describes the contribution of electric charge Q to the system. The charge affects the electromagnetic field around the black hole and is an important factor in understanding the interaction of black holes with charged particles in extreme environments [68]. Fifth, $[6a^2\pi/R^2]dJ$ shows the effect of angular momentum a on the black hole system. Angular momentum contributes to the rotational dynamics of the black hole, affecting the gravitational properties and structure of the surrounding space-time [69]. $[14.2\pi^2 r^2 R^6 \Lambda^2]d\rho$ involves a cosmological constant ($\Lambda$). A constant related to dark energy or cosmological vacuum effects that affects the evolution and behavior of black holes in the broader of cosmology [70] [71] [72].

*Hawking Temperature Interaction with Internal Variables*

*1. Area at hawking temperature*

In equation (28) for (dA), the main contributions of the radius (r) and mass (M) to the Hawking temperature are depicted as $3(r^2 + M)/4\pi R^2$ indicating that the larger the radius and mass of the black hole, the higher the effective temperature emitted [73]. The second term, $\frac{1}{2\pi}\{(3rM/R^2) + r\Lambda(r - M) + \pi r(r - M)\}$, describes a correction that considers the influence of the cosmological field ($\Lambda$) and the complex interaction between the radius and mass of the black hole [74]. The third term, $0.333r^2R^2$, is a non-linear correction that explains the effect of spacetime geometry on the Hawking temperature [75]. The variation of the value of the cosmological parameter ($\Lambda$) in the numerical analysis explains that the Hawking temperature is very sensitive to the cosmological field conditions around the black hole [76].

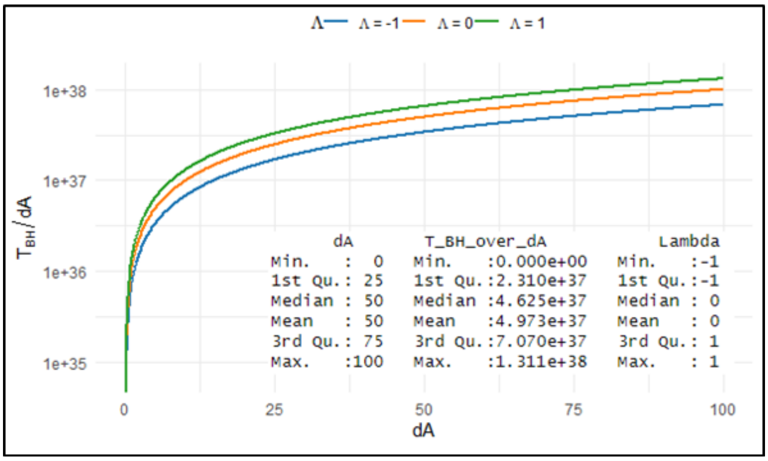


***Fig 1.*** *Variation of Modified Hawking Temperature with Area (dA) and Cosmological Parameters* ($\Lambda$)

Figure 1 is a visualization of Hawking temperature against area (dA) showing the thermal characteristics of a black hole, where the effective temperature varies with the observed area in accordance with Hawking radiation theory [77]. The logarithmic scale in the plot illustrates the wide range of values in the black hole phenomenon, although it should be noted that the logarithmic transformation can produce infinite values in the analysis [78].

*2. Charge change at Hawking temperature*

The equation shows that the Hawking temperature of a black hole is directly proportional to the square of the black hole's charge, and also depends on the square of the black hole's horizon radius and external radius (R) [79]. In practical implementation, values can be used $(r = 10^6)$ and $(R = 10^6)$ which is a commonly used scale in black hole studies. Changes in (Q) (0, 0.5, and 1) affect the value of the $(T_{BH}/dQ)$, which can be observed from the image below:

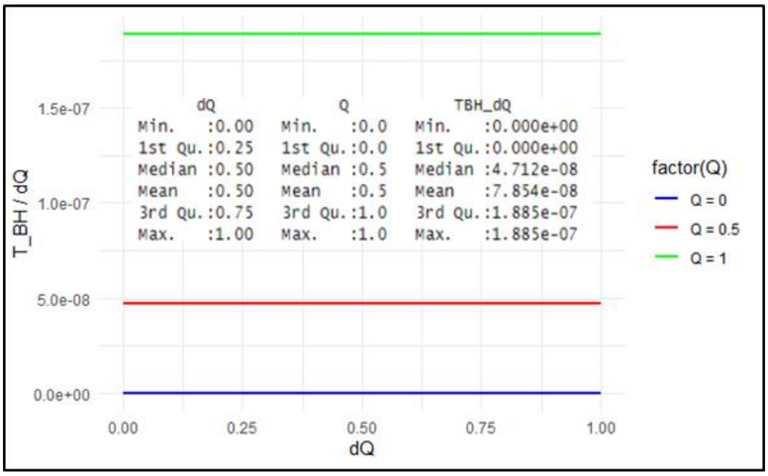


***Fig 2.*** *($T_{BH}/dQ$) for Different Q Values*

In Figure 2, the x-axis (dQ) describes the variation of the charge change (Q), the y-axis shows the value of the ($T_{BH}/dQ$). The different colored lines represent different values of (Q), with blue for (Q=0), red for (Q=0.5), and green for (Q=1). The Figure 2 visualizes how the Hawking temperature relative to the black hole charge changes, highlighting the different impact different charges have on the Hawking process [12].

*3. Change in angular momentum at hawking temperature*

In equation ($T_{BH}R^2 = dJ6a^2\pi$)is the relationship between the Hawking temperature and the change in angular momentum (dJ) of the black hole [80]. (a) shows the effect of black hole rotation on Hawking temperature. The larger the value of (a), the larger the contribution of rotation to the Hawking temperature [81]. (R) is the radius of the black hole which affects its gravitational strength and in this case is in the denominator, so the larger (R) is, the Hawking temperature will tend to decrease [82]. A change (dJ) in angular momentum causes a proportional change in Hawking temperature [83].

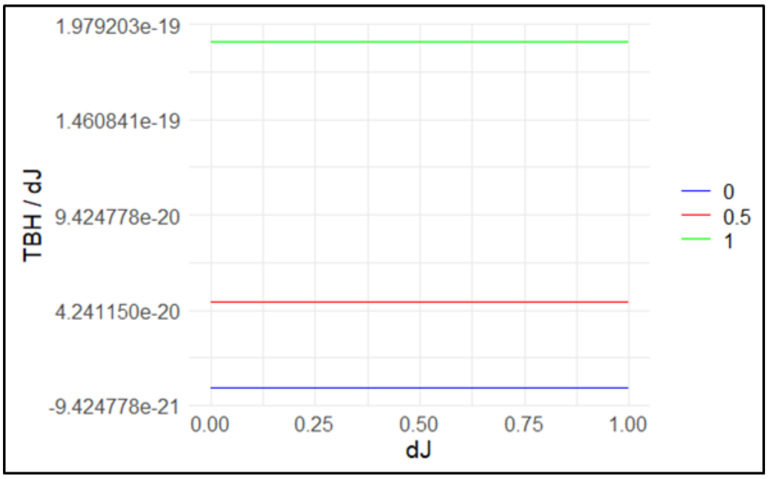


***Fig 3.*** *Variation (a) in $T_{BH}/dJ$*

The blue, red, and green lines represent the variation of (a) values (0, 0.5, 1) in the relationship of ($T_{BH}/dJ$), which in Figure 3. The lines illustrate how the Hawking temperature changes with changes in angular momentum, highlighting the significant differences between non-rotating and rotating black holes. The decrease in the curve with increasing (dJ) shows that the impact of angular momentum on Hawking temperature can be mathematically measured and understood in black hole physics [84].

*4. Change in dρ at Hawking temperature*

Figure 4 shows that $(T_{BH}/d\rho)$, which is the ratio of Hawking temperature to change $(d\rho)$, increased significantly when $(d\rho)$ increases. This indicates that larger changes in distance can result in larger changes in the observed Hawking temperature [85]. The color variations in the figure represent different Lambda values. The Lambda value controls the effect of cosmological parameters on the Hawking temperature and the observed change in the value of $(T_{BH}/d\rho)$ [86]. Figure 4 shows higher values of Lambda (Lambda = 1), there is a greater increase in $(T_{BH}/d\rho)$ with improvement $(d\rho)$, compared to a lower Lambda (Lambda = 0.25).

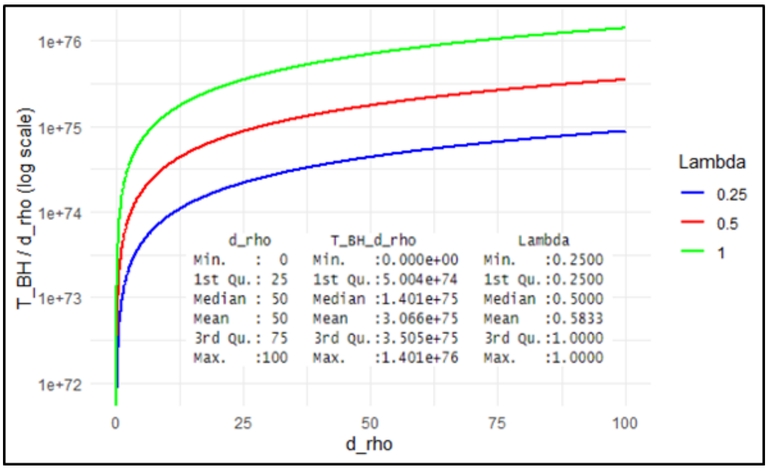


***Fig 4.*** *$(T_{BH}/d\rho)$ vs $(d\rho)$ for Different $(\Lambda)$ Values (log scale)*

Figure 4 can be interpreted as the response of the black hole to changes in distance and the influence of cosmological parameters. For example, a higher Lambda indicates more dark energy in the cosmological model, which can affect black hole properties such as the Hawking temperature observed from different distances [87].

## DISCUSSIONS

Black hole thermodynamics places great emphasis on the relationship between changes in physical parameters such as horizon area, electric charge, angular momentum, and vacuum energy with changes in black hole mass [88]. The thermodynamic equation (1) describes how the change in black hole mass (dM) depends on changes in horizon area (dA), electric charge (dQ), angular momentum (dJ), and vacuum energy (dρ). Each parameter has a specific definition and each makes a unique contribution to the change in black hole mass. Key components such as surface gravity $(\kappa)$, electric potential (Φ), angular velocity (Ω), and thermodynamic volume (V) play an important role in influencing the properties of black holes through their interaction with the mass and energy of the system [89].

The first law of thermodynamics for black holes (7) states that the energy change in the system (dE) is the result of the added thermal energy (T dS) minus the work done by the system (P dV) [90]. The change in the mass of the black hole (dM) can be explained as a response to the change in the total energy of the system. The fundamental relationship between mass (M) and energy (E) in general relativity, as described by Einstein's equation (8), confirms that the change in black hole mass (dM) is a direct result of the change in energy (dE) in the system [91]. The use of natural units in the normalization (13) makes it easier to interpret the equation to understand the relative contribution of each physical parameter to the change in black hole mass. The entropy (S) of the black hole, which is related to the horizon area (A) through the law of thermodynamics (22), which explains how variations in entropy affect changes in the black hole's thermal energy. The Hawking temperature (TH) (24) reinforces this relationship by showing how the change in thermal energy (dQ) can be calculated from the change in black hole entropy.

Equation (29) modifies the traditional Hawking temperature to include key parameters such as the black hole radius (r), mass (M), electric charge (Q), angular momentum (a), and cosmological constant ($\Lambda$). Each parameter contributes uniquely to the thermal entropy and dynamics of the black hole.

The contributions of radius (r) and mass (M) (Equation 29) underscore the role in shaping the thermal entropy of black holes. Larger values of (r) and (M) lead to higher thermal entropy, reflecting the greater complexity and thermal nature in the black hole's space-time dynamics. Influence of cosmological parameters ($\Lambda$) in Equation (29) introduces corrections that deepen our understanding of quantum field interactions near the black hole event horizon. The corrections highlight the complex relationship between black holes and the quantum fabric of space-time, going beyond classical gravitational theory. Equation (29) incorporates non-linear effects that deviate significantly from classical predictions. The non-linear corrections reveal the dynamical and structurally complex nature of black holes, suggesting interactions with quantum gravitational phenomena that challenge traditional astrophysical models. The role of electric charge (Q) in Equation (29) emphasizes its impact on the electromagnetic field surrounding the black hole. The parameter is crucial to understanding how the black hole interacts with charged particles and the electromagnetic environment, which affects the observed Hawking temperature. Angular momentum (a) contributes to the rotational dynamics of the black hole, affecting its gravitational properties and the structure of the surrounding space-time. Equation (29) illustrates how variations in angular momentum affect the Hawking temperature. On the internal variables, Equation (28) describes how the Hawking temperature varies with the black hole area (dA) and cosmological parameters ($\Lambda$).

The relationship depicted in Figure 1 visualizes these variations, showing how the observed temperature changes with area, in line with the theoretical expectations of Hawking radiation theory. The logarithmic scale in Figure 1 illustrates a wide range of temperature values, although caution is needed due to the potentially infinite values in the analysis. Regarding the black hole charge (Q), changes to Equation (29) show its direct impact on the observed Hawking temperature, as depicted in Figure 2. Different values of charge affect the temperature dynamics, highlighting the importance of charge in understanding black hole thermodynamics and radiation processes. Similarly, variations in angular momentum (a), as illustrated in Figure 3, affect the Hawking temperature differently for rotating and non-rotating black holes. The curves in Figure 3 show how different values of angular momentum change the temperature dynamics, emphasizing the role of rotation in black hole physics. Figure 4 illustrates the relationship between ($\rho$) (Hawking temperature changes in (r) and cosmological parameters ($\Lambda$). ($\Lambda$) higher indicates stronger dark energy effects, which affect the temperature response to distance changes. Figure 4 clarifies the impact of cosmological conditions on the observed Hawking temperature variations.

## CONCLUSION AND SUGGESTION

Study reveals the complexities of black hole thermodynamics in depth, highlighting the essential relationship between changes in horizon area, electric charge, angular momentum, and vacuum energy with changes in black hole mass. Basic thermodynamic equations illustrate how each parameter uniquely contributes to black hole dynamics, with entropy, Hawking temperature, and cosmological parameters playing a central role in deepening understanding of the properties. Non-linear corrections in the model show complex interactions with quantum and cosmological fields, challenging conventional paradigms in black hole physics and providing a solid basis for continued research.

Based on the results, there are several promising directions of further research to be explored. First, it is important to further develop the thermodynamic modeling of black holes to deepen the understanding of how changes in parameters such as electric charge, angular momentum and vacuum energy specifically affect the thermodynamic properties of black holes. In-depth studies of the effects of cosmological parameters in the black hole thermodynamic equations are also needed, including how variables affect the thermal processes and entropy of black holes.

Deeper research into the quantum aspects of black hole thermodynamics may reveal a deeper connection between quantum field theory and the thermodynamic properties of black holes, especially in the non-linear corrections that have been described in this study. Empirical and observational studies using actual data from astronomical observations are also needed to test and validate the predictions of black hole thermodynamic theory.

Further research could explore the implications of these results for alternative theories of gravity or modifications of general relativity, particularly in relation to cosmological consequences and broader quantum phenomena. It is important to investigate how variations in physical parameters such as electric charge and angular momentum can affect the Hawking temperature predictions as well as the thermal properties of black holes in general.

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