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HOW STUDENTS MAKE MODEL AND PROMOTE PRODUCTIVE STRUGGLE IN SOLVING MATHEMATICAL PROBLEMS

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Abstract. Creating mathematical models is a crucial factor in solving problems. This research aims to provide a detailed description of students' difficulties in creating mathematical models in solving contextual problems related to ratio and linear equations. The study adopts a qualitative approach with a phenomenological hermeneutic design. The research subjects consist of 52 eighth-grade students from three schools in Bandung. Data collection techniques include tests and interviews. Before usage, the instruments were validated by experts, particularly in terms of content, construct, and appearance. Data analysis involves identifying meanings, determining credibility, interpreting data, identifying themes, and compiling the final report. The research draws four conclusions. First, students' difficulties in creating mathematical models include a failure to understand the situation/mathematics conceptually, an inability to identify relevant variables, creating mathematical models without involving context, failing to produce relevant visualizations, failing to use adequate formulas, and failing to answer questions with the assistance of mathematical results. Furthermore, they fail to adjust (temporarily) their results to the actual situation. The main factors influencing students' difficulties in creating mathematical models are their lack of understanding of mathematical concepts, their inability to apply these concepts, and their limited practical experience in mathematical modeling. Their limited practical experience in mathematical modeling made the iconic model ability lower than the symbolic model ability.

Keywords: Mathematical Model, Productive Struggle, Problem-Solving

I. INTRODUCTION

Solving problems in mathematics is one of goals in learning (NCTM, 2000). All problems in everyday life can be solved through mathematics, because many problems require mathematical thinking (Herman, 2020). Mathematics can be a tool to describe, analyze, and predict events in various real-world contexts (Verschaffel et al., 2002). However, the fact showed that learning mathematics is not always interesting for students. Even students who has

interest to mathematics, they still face many difficulties, for example in word problems and mathematical modelling process. (Herman, 2020; Maskar et al., 2022).

Mathematical modelling task is to translate between reality and mathematics (Blum 2015). Mathematical modelling is an activity that needs understanding abilities, mathematical connections, non-mathematical competencies, and high mathematical knowledge (Blum 2015). The ability to create a mathematical model is part of the problem-solving process, because when we created mathematical

model its mean we create planning to solve problems (Polya 1957; Yimer and Ellerton 2010).

The issue in problem-solving has not been completely resolved. Research results still indicate difficulties that students unable to interpret the issues into mathematical models (Sulistiowati, Herman, and Jupri 2019). Formulating the problems is significantly difficult for students (Herman, 2020).

This is supported by the findings of several other researchers that the difficulty in formulating situations was in the use of variables, and mathematical connections, and finding the patterns within the problems (Asempapa & Sturgill, 2019; Cevikbas et al., 2022; Edo et al., 2013; Greefrath et al., 2013; Greefrath & Vorhölter, 2016; Hankeln et al., 2019; Hidayat et al., 2021; Kassel et al., 1990; Melani et al., 2023; Mingke & Alegre, 2019).

Based on the results of the research conducted, the productive struggle of students is a crucial role in the performance of the process of creating mathematical models (Melani et al., 2023). Productive struggle is highly essential in mathematics class, helping students to think deeply. Being able to apply this ability is good for students as problem solvers but transforming the classroom into a productive struggle situation is not an easy task. (NCTM, 2017). Therefore, it seems important to analyze how students create models and promote productive struggle in solving mathematical problems.

II. METHODS

This research uses a qualitative approach with a case study method. Qualitative research explores naturally occurring events. (Yin, 2015). The researcher aims to explore more about students' ability to create mathematical models using an individual case. **the specific aim in evaluating students' abilities in using iconic and symbolic mathematical models.** The study was conducted in one high school in the Bandung West Java, Indonesia. The subjects of this research were 52 students and 2 teachers. The research instruments for this study were tests, interviews, and observations.

There were four principles in this research credibility, transferability, dependability, and confirmability (Yin, 2015). The principle of credibility ensures that this research accurately collects and interprets data, ensuring that the findings and conclusions faithfully reflect and represent the researched object. To adhere to this principle, the researcher extends the observation period, increases diligence, seeks diverse data, and even when faced with contradictory data, employs supportive reference materials to validate the discovered data.

The second principle, transferability, recognizes the uniqueness of local conditions, which can hinder the generalization of research findings to other situations. (Lincoln & Guba, 1985). To adhere to this principle, the researcher meticulously outlines the research results in a detailed, clear, and systematic manner.

Thirdly, dependability, by definition, denotes reliability, trustworthiness, or accountability. To uphold this principle,

the researcher provides a comprehensive summary of the entire research process, encompassing a detailed record of field activities. The final principle is confirmability. Research meeting this criterion implies that the results and research processes are subject to verification for accuracy. To fulfill this requirement, the researcher engages in observations and conducts in-depth interviews with the research subjects.

The indicators of mathematical representation or mathematical models used by the researcher are as follows.

TABLE I
 INDICATORS OF MATHEMATICAL MODELS

Indicator	
Iconic	Symbolic
Students can translate contextual problems into pictures, graphs, and diagrams (Wingard-nelson, 2005)	Students can translate contextual problems into numbers or algebraic equations (Wingard-nelson, 2005)

The subjects of this research were secondary school students, so the focus is on iconic and symbolic models.

III. RESULT

The ability of students in making mathematical models can be evaluated through the scoring obtained during the test. A total of 52 students from Junior High Schools in Bandung participated as respondents in this study. The test presented consisted of ten questions. The test was designed to cover non-routine contextual problems with varying levels of difficulty.

Questions 1 to 10 researchers analyzed using score guidelines 1-4 used by the Programme Student Assessment (PISA). The results of the mathematical modelling ability test are described in Table III below.

TABLE II
 TEST RESULTS FOR THE ABILITY TO MAKE MATHEMATICAL MODELS BASESD ON TYPES OF MODELS

Type of model	Ideal score	average
Iconic	100	39,3
Simbolic	100	40,5

The table above shows that the ability to use symbolic models is more dominant than iconic models.

The difference in scores between iconic and symbolic models in mathematics education is influenced by several important factors, including the role of the teacher, students' experiences, individual abilities, and specific contexts. The role of the teacher is crucial in this regard (Leong et al., 2015; Sundari & Fauziati, 2021). The teaching methods used by teachers in introducing and teaching iconic models and symbolic models (abstract representations) can determine the level of students' understanding and skills. Based on research findings, observations indicate that the lesson plans designed by teachers do not demonstrate the teaching of model-making techniques, especially iconic models. Teachers tend

to use symbolic models to teach their students. This shows a lack of focus on teaching iconic models, which can affect students' understanding of mathematical concepts taught visually.

Students' previous experiences with iconic and symbolic models also affect their skills in mathematics (Leong et al., 2015; Sundari & Fauziati, 2021). Based on interview results, students are not familiar with iconic models and are just learning about them. Students who practice more frequently with one type of model are more likely to understand and use that model, which ultimately affects their scores.

The researchers grouped students into two categories, that we can see in Table 7.

TABLE III
 TEST RESULTS FOR THE ABILITY TO MAKE MATHEMATICAL MODELS BASED ON COMPLETENESS

Score	Number of students	Category
score > 70	2	Complete
score ≤ 70	50	Incomplete

Based on the data on students' achievement abilities above, it can be seen that only two people completed the problem of making mathematical models, meaning that there were difficulties faced by students so that they did not reach the minimum score. In this section, we will describe the findings regarding students' difficulties in making mathematical models which are contained in Table V.

TABLE V
 STUDENTS' DIFFICULTIES IN MAKING MATHEMATICAL MODELS

Category (modeling steps)	Subcategory (modeling sub-processes)
Forming real-word model	1. Fail to understand the context
	2. Fail to identify relevant variables
	3. Calculating without including the context
	4. Fail to identify relevant variables
Working mathematically	5. Fail to use adequate formulas
Interpreting	6. Fail to answer the question with the help of the mathematical results
Validating	7. Failing to adjust (temporary) results to the actual situation

Based on what has been described above, the biggest difficulty for students is answering questions with the help of mathematical results.

This statement is illustrated by the students' answers in question number 3. Students are asked to calculate the number of tons of coffee exported to Algeria in 2021 if it is known that the ratio of the number of coffee exports to Algeria and Romania in 2021 is 1: 2. In 2022, Indonesia will increase the amount of exports to Algeria is 15 tons more than in 2021, so the ratio is 2:1. To find out the number of tons of coffee exported to Algeria in 2021, students are asked to express the problem in mathematical model. The

following are the answers that students have found, as shown in Fig. 1.

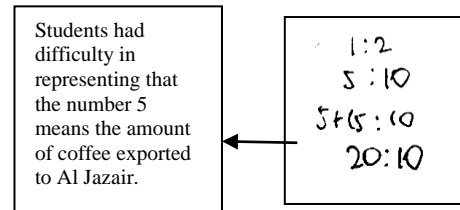


Fig. 1 Student's answer in number 3

From the illustration above, it can be seen that students succeeded in converting contextual problems into the form of mathematical expressions. However, they still faced difficulties. An interview was conducted with the subject to confirm the student's answer. The interview results can be seen as follows.

Researcher : The mathematical model you made is very good, you are the only student from the number of my research who was able to make a mathematical model on this problem correctly.

S11 : sure ma'am? Thank you (with a happy expression)

Researcher : Yes, that's right, but you haven't got the answer to how many tons were exported to Al Jazair in 2021?

S11 : Well, honestly this question is the most difficult, and I am confused about the answer.

While students have the capability to build mathematical models, but to get the final answer from the model is still a challenge. This could indicate that students need more help or more practice in applying the mathematical models they have built to find the final solution to a problem.

The results of the interview process show that the learning process has less mathematical model activity. The teacher explained that many students still have difficulties in understanding mathematics. Teacher admits that he has difficulty in facilitating other students who already have a good understanding of mathematics to move on to the next level, namely mathematical models.

Student Difficulties in Identifying Relevant Variables and Using an Appropriate Formula

Variables has a significant role in mathematical contexts. A variable refers to a symbol or letter that represents a score that can change or vary. Students' difficulty in using relevant variables can be seen when students are asked to determine the number of pens in one box if it is known that the total of two boxes and two pens is 30 pieces. Their answers can be seen in the picture 2 below.

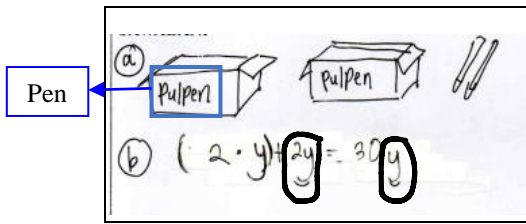


Fig 2. Student's answer to Problem Number 1

Students make the picture model correctly, but the variable y in the coefficients 2 and 30 is irrelevant, the mathematical model should be a constant. It's should be 2 and 30 only without "y".

Researcher : Are you sure about the equation you made? (while pointing to the student's answer).

S22 : a little bit sure ma'am

Researcher : Try to pay attention anymore!

S11 : I guess it's right ma'am.

Students did not fully realize the errors in the mathematical models they made, even when the researcher tried to give scaffolding guidance. This situation indicates that the inaccuracies were not caused by students' lack of accuracy, but because students were not able to make relevant variables.

Student Difficulties in Model Validation

The "looking back" stage is an essential factor in doing problems, especially in the context of creating mathematical models (Blum, 2015). Validation is an important stage to ensure accuracy, to avoid errors, to explore alternatives, and to maintain openness to possible errors. However, not all students have the habit of checking the answers they have made. This was confirmed when students answered a question about the height of two people if the difference and sum of their heights were known, as in the picture below.

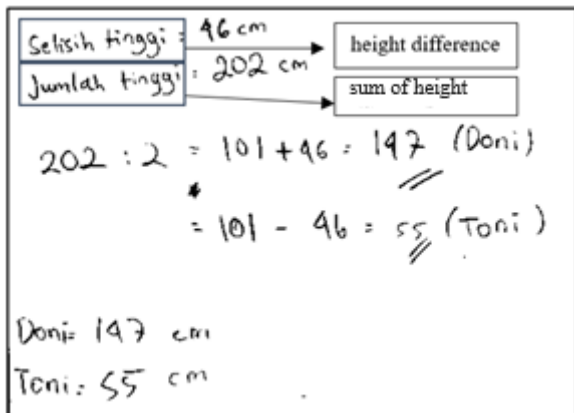


Fig 3. Student's Answer to Problem Number 8

The student's answer fulfills the requirements for the number of heights, but the difference is 92, which means it's wrong. The following is an excerpt of an interview with a student related to the student's reasoning in answering the question.

Researcher : Are you sure about the answer? (while

pointing to the student's answer).

S32 : Sure Mom.

Researcher : Have you checked the correctness?

S32 : Not yet ma'am.

Researcher : Why didn't you check it again?

S32 : I didn't think of it, ma'am.

Researcher : Try to explain the process of your answer!

S32 : I halved the number, then I added the result of the division, and subtracted the same number to get Toni's height.

Researcher : Let's check the answer together. How many conditions are met in this problem?

S32 : (While reading again), there are two ma'am, the difference is 46 and the sum must be 202. The sum of their height is correct ma'am.

Researcher : The difference is 92, which means it's wrong.





From this statement, it appears that students skipped the validation phase of the model they created, so they did not correct the errors in creating the model. The missing of students in the model validation process can have a serious impact on the quality of the model, as the validation phase is a key step to identify and address potential errors or inaccuracies in the model.

Students' Difficulties in Making Relevant Visualizations

Visualization is closely related to the process of creating models in the form of pictures, figures, graphs and diagrams. This ability is no less important than the skills previously described, but this relevance is not always in line with the reality experienced by students in their daily lives. The research findings provide support for this statement, with the results of students' answers when asked to describe contextual situations in the form of drawings or other iconic representations.

It is known that the ratio of coffee exports to Algeria and Romania in 2021 is 1:2. In 2022, Indonesia increased the amount of exports to Algeria by 15 tons more than in 2021 so that the ratio became 2:1. To find out the number of tons of coffee exported to Algeria in 2021. State the above situation in the form of a picture!

Here are the expected answers:

Year/country	Al Jazair	Romania	ratio
2021			1:2
2022			2:1

$$\frac{15}{3} = 5$$

So the coffee exported to Algeria in 2021 is 5 tons.

Students who face difficulties in making visualizations are seen through the image presented below.

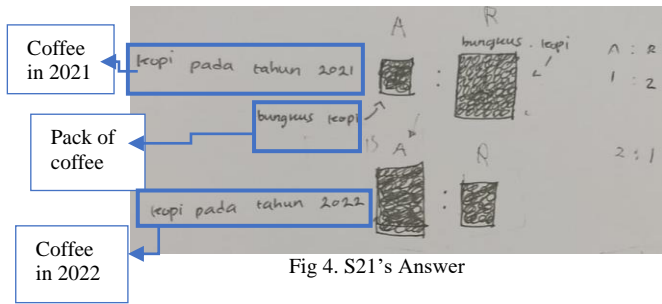


Fig 4. S21's Answer

From the aforementioned response, it is evident that the student has yet to master the skill of proficiently crafting a mathematical pictorial model. The student's comprehension appears limited to the theoretical understanding of the term "comparison," without a corresponding ability to effectively employ the concept in resolving presented problems. To validate the student's response, an interview was carried out with the participant, and the outcomes of this interview are detailed below.

- R : Do you believe the ratios you've formulated are appropriate for the question's context?
 S : I'm not certain, miss. It's challenging to depict the situation in the form of a diagram.
 R : However, you comprehend the question's meaning, correct?
 S : Yes, I understand the question, miss. But I'm perplexed about how to visually represent it.

Based on the results of answers and interviews with students, students experience difficulty in visualizing information and identifying relationships between elements in constructing visual mathematical models.

Students' Difficulties in Understanding Situations/Mathematics Conceptually

When given problems involving percentages, students provide answers that do not correspond to the concept of percentages or are unable to apply their knowledge correctly. The researcher provides contextual problems regarding a school that holds an end-of-semester final exam, where 25% of students achieve scores above the minimum criteria. How many students took the exam that day if 300 students scored above the minimum criteria?

The following are the answers of students who have difficulties.

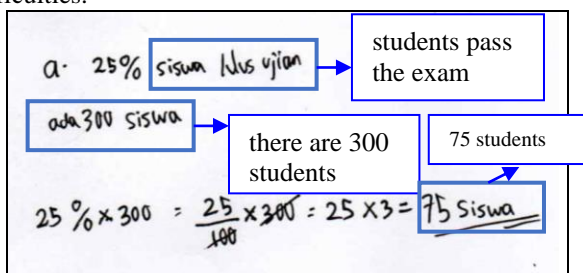


Fig 5. S25's answer to Problem Number 4

Based on the picture above, it can be seen that students have not fully understood the concept of percentage.

Students have difficulty in connecting the percentage with the actual number or score, so the answers given are not in accordance with the context of the problem. To confirm students' answers, an interview was conducted with the subject. The interview results can be seen as follows.

- Researcher : Can you explain the reason for getting this answer?
 S21 : That's 25% ma'am, just multiply it by 300, the result is 75 students
 Researcher : Are you sure you just multiplied it?
 S21 : Yes ma'am, it's a percentage.
 Researcher : Try to read the question again, the meaning of 25% should be from the total students or from 300 students?
 S21 : Uh yes ma'am, it should be from the total students.
 Researcher : So, are you sure the answer is 75 students?
 S21 : I'm not sure, ma'am. Sorry, I didn't read the question correctly, I wasn't careful, and I don't really understand this fraction or percent problem.
 Researcher : Try to calculate it again.
 S21 : Yes, ma'am.

This interview reflects that students have difficulty in understanding the concept of percentage and lack of accuracy in reading the questions. Students' openness in admitting their lack of understanding and skills creates an opportunity for researchers or teachers to provide additional guidance in understanding the concept of percentages and improving their skills in reading questions carefully. This is in accordance with research that students have difficulty determining the right solution strategy and linking concepts to double-check the answers obtained (Mefiana et al., 2023; Pebrianti et al., 2023).

A. Factors of Students' Difficulty in Making Mathematical Model

The difficulties faced by students are always related to specific factors. Based on the results of interviews with teachers and students, additional information about the factors of students' difficulties in making mathematical models is obtained and summarized in the table V below.

TABLE V
 FACTORS OF STUDENTS' DIFFICULTY IN MAKING MATHEMATICAL MODELS BASED ON COMPLETENESS

Factors of Students' Difficulty in Making Mathematical Models	
Internal	<ol style="list-style-type: none"> 1. Students lack interest in practicing contextual and non-routine problems. 2. Students are less critical in solving problems. 3. The students are not able to try repeatedly to obtain the expected mathematical model. 4. Lost learning 5. Weak prior knowledge of students. 6. Students lack confidence when

	facing challenging problems. Students seldom allocate their time for studying mathematics.
External 8.	Lack of practical experience in making models with various types and methods.

Based on the findings of this research, we will elaborate about the factors of Students' Difficulty in Making Mathematical Models. the first one, students' is not interest in practicing contextual and non-routine problems can hinder students' ability to apply mathematical concepts to real situations, reducing their ability to make contextual and relevant models. Second, students' lack of criticality in solving problems can hinder the process of making mathematical models, as students may not analyze carefully and formulate appropriate solutions.

Third, students' inability to try repeatedly can impede the development of perseverance and skills in making mathematical models. Limited exploration of solutions can restrict their understanding. Fourth, lost learning experiences make difficulties for students to grasp higher-level mathematical concepts, which are necessary for making more complex models. Fifth, limited prior knowledge be a major obstacle in formulating mathematical models. Lack of fundamental understanding can hinder students' ability to apply these concepts.

Sixth, students' lack of confidence in facing challenging problems can affect their ability to make complex and creative mathematical models. Seventh, the lack of allocated time for learning mathematics can hinder students' understanding of mathematical concepts and skills in making mathematical models. Furthermore, a lack of practical experience in making models with various types and methods can make students struggle to connect mathematical theory with real-world contexts, reducing their ability to make applicable models.

In addition, during the research process, differences in students' characteristics when making mathematical models were revealed, particularly regarding the habit of dealing with challenging and non-routine problems. Specific cases indicate that only one student out of the entire group tends to handle problem number 10 due to the habit of learning at a tutoring place that emphasizes solving challenging problems. On the other hand, students is not familiar with non-routine problems tend to show a level of hesitancy, even after attempting, but without productive struggle. From these results, it is evident that the lack of interest in non-routine problems has a significant impact on students' performance in facing the task of making mathematical models.

Examples of the factors mentioned above.

Problem : Mother has $\frac{1}{2}$ kilograms of sugar. Every day, she uses $\frac{1}{6}$ kilograms of that sugar. To determine how many days the sugar will be used up.

- a) state the situation above using a diagram!
- b) How many days will it take until the sugar runs out?

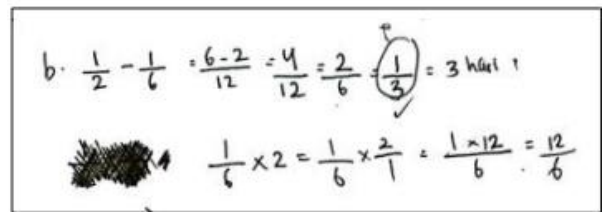


Fig 6. Student answers that indicate lost learning

The student successfully started with the correct steps in forming a mathematical model, using the approach of half minus one divided by six. Initially, the final answer, 3 days, seemed correct at first glance. However, the researcher then took the initiative to re-examine every detail of the answer. A deeper analysis revealed that the mathematical model created by the student was incomplete.

The student subtracted once to obtain the answer, but the final answer turned out to be one-third. The student then forcibly rounded the result to 3, although one-third is not equivalent to three. This indicates that the student did not fully understand the meaning of the concept of one-third and also did not grasp the correct steps in designing the mathematical model, resulting in lost learning in this case. This is supported by the teacher's statement that after Covid-19, students experienced significant lost learning, resulting in incomplete understanding of many topics.

IV. DISCUSSION

Students' ability to make mathematical models shows scores below the minimum score. This is in line with research which states that students are still low, because many students are not able to solve problems well in each aspect (Samosir et al., 2020). Teachers must be perfect in guiding students, including providing practical experience regarding various representations such as enactive, iconic and symbolic. This experience must pass well, in the order of enactive, iconic and symbolic. This aims to build students' concept understanding to be meaningful and in accordance with students' cognitive stages from concrete to abstract (Leong et al., 2015; Oakley & Lecturer, 2004).

Enactive representation is concerned with learning by doing (Leong et al., 2015). Iconic is a series of summarized images or graphics that represent a concept without fully defining it (Leong et al., 2015). Symbolic representation is a set of symbolic or logical propositions drawn from a symbolic system governed by rules (Leong et al., 2015). Mathematical symbols are used to represent mathematical objects such as: numbers, functions, and limits, as well as operations such as addition and multiplication (Heritage & Niemi, 2006). Symbolic representation focuses on symbolic notation and includes the use of variables and formulas. Symbolic representations such as equations, algebraic equations, algebraic expressions, and formulas (Heritage & Niemi, 2006).

The results showed that the symbolic math model was better than the iconic math model. However, the symbolic

ability is still below the minimum score. Therefore, it is necessary to improve this ability through more careful and systematic stages. Students are better off being taught again the process of iconic representation; it is preferable and will be more beneficial. This model helps students understand mathematical concepts thoroughly and relevantly (Bronkhorst et al., 2021). Iconic representation is effective for students to know concepts that can be seen with the naked eye (Meiers & Trevitt, 2010).

To tackle this challenge necessitates a more focused approach to learning, incorporating engaging and contextually relevant learning strategies, enhancing critical skills, and providing adequate support to bolster self-confidence (Melani & Herman, 2023). These conclusions provide a strong foundation for designing effective solutions to improve students' ability to formulate mathematical models when facing the problem-solving process. To implement these findings, a number of practical suggestions can be adopted in the context of mathematics learning. Teachers need to understand the concept of Zone of Proximal Development (ZPD) well and be able to select problems that are appropriate to students' ability levels when designing lessons. The integration of problem-based learning models should be the focus, with an emphasis on selecting challenging problems that are appropriate to students' ZPD. Developers of teaching materials or modules also need to ensure that materials follow a structured learning flow, include evaluation of student progress, provide predictability of student responses, and offer effective guiding questions.

Holistic Recommendations

The Ministry of Education is recommended to introduce and formulate policies for the implementation of the enactive-iconic-symbolic method, similar to the government's initiative in promoting problem-solving models or contextual learning to be applied in every school.

The Enactive-iconic-symbolic (EIS) method combines three primary types of representation—enactive (through direct action or experience), iconic (via visual or graphical representation), and symbolic (using abstract symbols or formal language). This approach offers several advantages in mathematics education. Firstly, it allows students to learn through direct experience, enhancing their understanding of concepts through practical engagement. Secondly, visual representations aid in providing concrete and intuitive understandings of mathematical ideas. Thirdly, the integration of symbolic representations supports students in grasping and applying concepts in abstract contexts. Overall, the EIS method provides a holistic approach to learning, accommodating various learning styles and fostering creativity in mathematical expression and interpretation (Leong et al., 2015). It has proven effective in enhancing students' mathematical skills by leveraging diverse forms of representation to deepen comprehension and facilitate comprehensive learning experiences.

V. CONCLUSIONS

The research results indicate that there is a deficiency in students' ability to construct mathematical models, with the majority of scores below the minimum score. These findings indicate the need for improvement and enhancement in students' mathematical modeling skills. Factors causing students' difficulties involve lack of interest in non-routine problems, lack of critical thinking in solving problems, lack of confidence in facing challenging problems, and lack of practical experience in making mathematical models. The use of symbolic models is more dominant than iconic models, although symbolic is often considered more abstract than iconic, the difference in average scores between students who use symbolic and iconic models can be influenced by several factors, namely the role of the teacher in delivering the material, the influence of students' previous experience, variations in students' individual abilities, and the specific context in which iconic understanding can be a determining factor in student success.

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No.	Representasi	Description
1.	Enactive	This representation is related to learning by doing (Leong et al., 2015). Students are expected to be able to represent mathematical situations concretely.
2.	Iconic	Students are expected to be able to create pictorial, graphical, and diagram models from mathematical situations (Leong et al., 2015; Wingard-nelson, 2005)
3.	Symbolic	Students can create mathematical symbols to represent mathematical objects such as numbers, functions, limits, and operations like addition and multiplication. (Heritage & Niemi, 2006), equations, algebraic equations, algebraic expressions, and formulas (Wingard-nelson, 2005).

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